NAPLAN Algebra Kick-Starter

Band 8 Path (Year 9)

Advanced Algebraic Concepts for Excellence

Welcome to Band 8 Algebra

This comprehensive guide is designed to elevate your algebraic thinking to Band 8 level for NAPLAN Numeracy. Band 8 represents advanced mathematical proficiency, requiring sophisticated problem-solving skills and deep conceptual understanding.

What You'll Master:

- Advanced equation solving techniques
- Quadratic expressions and functions
- Systems of equations
- Complex algebraic manipulation
- Real-world algebraic modelling

Band 8 Characteristics:

- Multi-step problem solving
- Abstract reasoning
- Pattern generalisation
- Mathematical communication
- Strategic thinking

Chapter 1: Advanced Equation Solving

Key Techniques for Band 8

Linear equations: $ax + b = cx + d$

Equations with fractions: $x/a + b/c = d$

Equations with brackets: $a(x + b) = c(x - d)$

Worked Example 1.1

Solve: $3(2x - 5) = 2(x + 7) - 1$

Step 1: Expand brackets: 6x - 15 = 2x + 14 - 1

Step 2: Simplify right side: 6x - 15 = 2x + 13

Step 3: Collect like terms: 6x - 2x = 13 + 15

Step 4: Solve: 4x = 28, therefore x = 7

Check: LHS = 3(14 - 5) = 27, RHS = 2(7 + 7) - 1 = 27 ✓

Practice Problems

1.1 Solve: 5(x - 3) = 3(x + 1) + 8

1.2 Solve: $\frac{2x + 1}{3} = \frac{x - 2}{2}$

1.3 A rectangle has length (3x + 2) cm and width (x - 1) cm. If the perimeter is 34 cm, find x.

Solutions

1.1: 5x - 15 = 3x + 3 + 8 → 2x = 26 → x = 13

1.2: Cross multiply: 2(2x + 1) = 3(x - 2) → 4x + 2 = 3x - 6 → x = -8

1.3: Perimeter = 2(3x + 2) + 2(x - 1) = 34 → 8x + 2 = 34 → x = 4

Chapter 2: Quadratic Expressions and Equations

Quadratic Forms

Standard form: $ax^2 + bx + c = 0$

Factored form: $a(x - p)(x - q) = 0$

Completing the square: $a(x - h)^2 + k = 0$

Worked Example 2.1

Factorise and solve: $x^2 - 5x + 6 = 0$

Step 1: Find factors of 6 that add to -5: -2 and -3

Step 2: Factorise: (x - 2)(x - 3) = 0

Step 3: Solve: x - 2 = 0 or x - 3 = 0

Solution: x = 2 or x = 3

Check: When x = 2: 4 - 10 + 6 = 0 ✓

Worked Example 2.2

Expand and simplify: $(2x + 3)^2 - (x - 1)^2$

Step 1: Expand $(2x + 3)^2 = 4x^2 + 12x + 9$

Step 2: Expand $(x - 1)^2 = x^2 - 2x + 1$

Step 3: Subtract: $4x^2 + 12x + 9 - (x^2 - 2x + 1)$

Step 4: Simplify: $4x^2 + 12x + 9 - x^2 + 2x - 1$

Answer: $3x^2 + 14x + 8$

Practice Problems

2.1 Factorise: $x^2 + 7x + 12$

2.2 Solve: $x^2 - 9 = 0$

2.3 Expand: $(3x - 2)(x + 4)$

2.4 A ball is thrown upward. Its height h metres after t seconds is given by $h = -5t^2 + 20t + 2$. When is the ball at ground level?

Solutions

2.1: $(x + 3)(x + 4)$

2.2: $(x - 3)(x + 3) = 0$, so $x = \pm 3$

2.3: $3x^2 + 12x - 2x - 8 = 3x^2 + 10x - 8$

2.4: Set $h = 0$: $-5t^2 + 20t + 2 = 0$. Using quadratic formula: $t \approx 4.1$ seconds

Chapter 3: Functions and Graphs

Function Notation and Properties

Function notation: $f(x) = ax + b$

Linear function: $f(x) = mx + c$ *(gradient m, y-intercept c)*

Quadratic function: $f(x) = ax^2 + bx + c$

Domain and range concepts

Worked Example 3.1

Given $f(x) = 2x - 3$, find:

a) $f(5) = 2(5) - 3 = 10 - 3 = 7$

b) $f(-2) = 2(-2) - 3 = -4 - 3 = -7$

c) Solve $f(x) = 11$: $2x - 3 = 11 \rightarrow 2x = 14 \rightarrow x = 7$

d) The gradient is 2, y-intercept is -3

Worked Example 3.2

Find the equation of the line passing through (2, 5) and (6, 13):

Step 1: Find gradient: $m = \frac{13 - 5}{6 - 2} = \frac{8}{4} = 2$

Step 2: Use point-gradient form: $y - 5 = 2(x - 2)$

Step 3: Simplify: $y - 5 = 2x - 4$

Answer: $y = 2x + 1$

Check: When $x = 6$: $y = 2(6) + 1 = 13$ ✓

Practice Problems

3.1 If $g(x) = x^2 - 4x + 3$, find $g(2)$ and $g(-1)$

3.2 Find the equation of the line with gradient -3 passing through (1, 7)

3.3 For what value of x does $f(x) = 3x + 2$ equal 20?

3.4 A mobile phone plan costs £25 plus £0.15 per minute. Write a function for the total cost $C(m)$ after m minutes.

Solutions

3.1: $g(2) = 4 - 8 + 3 = -1$; $g(-1) = 1 + 4 + 3 = 8$

3.2: $y - 7 = -3(x - 1) \rightarrow y = -3x + 10$

3.3: $3x + 2 = 20 \rightarrow 3x = 18 \rightarrow x = 6$

3.4: $C(m) = 25 + 0.15m$

Chapter 4: Systems of Equations

Solution Methods

Substitution method

Elimination method

Graphical interpretation

Applications in real-world problems

Worked Example 4.1 - Substitution Method

Solve the system:

$2x + y = 7$ ... (1)
$x - y = 2$ ... (2)

Step 1: From equation (2): $x = y + 2$

Step 2: Substitute into (1): $2(y + 2) + y = 7$

Step 3: Expand: $2y + 4 + y = 7$

Step 4: Solve: $3y = 3$, so $y = 1$

Step 5: Find x: $x = 1 + 2 = 3$

Solution: $x = 3, y = 1$

Worked Example 4.2 - Elimination Method

Solve the system:

$3x + 2y = 16$ ... (1)
$5x - 2y = 8$ ... (2)

Step 1: Add equations (1) and (2): $8x = 24$

Step 2: Solve: $x = 3$

Step 3: Substitute back: $3(3) + 2y = 16$

Step 4: Solve: $9 + 2y = 16$, so $y = 3.5$

Solution: $x = 3, y = 3.5$

Practice Problems

4.1 Solve: $x + 2y = 8$ and $3x - y = 1$

4.2 Solve: $2a + 3b = 13$ and $a - b = 1$

4.3 The sum of two numbers is 25. Their difference is 7. Find the numbers.

4.4 Adult tickets cost £12 and child tickets cost £7. If 35 tickets were sold for £340, how many of each type were sold?

Solutions

4.1: $x = 2, y = 3$

4.2: $a = 4, b = 3$

4.3: Let x + y = 25 and x - y = 7. Numbers are 16 and 9.

4.4: Let a = adults, c = children. a + c = 35, 12a + 7c = 340. Solution: 15 adults, 20 children.

⚖️ Chapter 5: Inequalities

Inequality Rules

Basic symbols: $<, >, \leq, \geq$

Addition/subtraction: preserve direction

Multiplication/division by positive: preserve direction

Multiplication/division by negative: reverse direction

Worked Example 5.1

Solve: $3x + 7 > 2x - 5$

Step 1: Subtract 2x from both sides: $x + 7 > -5$

Step 2: Subtract 7 from both sides: $x > -12$

Solution: $x > -12$

Check: Try x = 0: 3(0) + 7 = 7, 2(0) - 5 = -5. Since 7 > -5 ✓

Worked Example 5.2

Solve: $-2x + 3 \leq 11$

Step 1: Subtract 3 from both sides: $-2x \leq 8$

Step 2: Divide by -2 (reverse inequality): $x \geq -4$

Solution: $x \geq -4$

Note: Direction reversed because we divided by negative number

Practice Problems

5.1 Solve: $4x - 3 < 2x + 9$

5.2 Solve: $-3x + 1 \geq 10$

5.3 Solve: $2(x - 3) > 3(x + 1)$

5.4 A taxi charges £3.50 plus £0.80 per kilometre. For what distances is the total cost less than £15?

Solutions

5.1: $2x < 12$, so $x < 6$

5.2: $-3x \geq 9$, so $x \leq -3$

5.3: $2x - 6 > 3x + 3$, so $-9 > x$, therefore $x < -9$

5.4: $3.50 + 0.80d < 15$, so $d < 14.375$ km

Chapter 6: Exponential and Growth Patterns

Exponential Laws

$a^m \times a^n = a^{(m+n)}$

$a^m \div a^n = a^{(m-n)}$

$(a^m)^n = a^{(mn)}$

$a^0 = 1$ (for $a \neq 0$)

$a^{(-n)} = 1/a^n$

Worked Example 6.1

Simplify: $3^4 \times 3^2 \div 3^3$

Step 1: Use addition law: $3^4 \times 3^2 = 3^{(4+2)} = 3^6$

Step 2: Use subtraction law: $3^6 \div 3^3 = 3^{(6-3)} = 3^3$

Step 3: Evaluate: $3^3 = 27$

Answer: 27

Worked Example 6.2 - Compound Interest

Problem: £1000 is invested at 5% per annum compound interest. What is the value after 3 years?

Formula: $A = P(1 + r)^n$

Given: P = £1000, r = 0.05, n = 3

Calculation: $A = 1000(1.05)^3$

Step 1: $(1.05)^3 = 1.157625$

Answer: A = £1157.63

Practice Problems

6.1 Simplify: $2^5 \times 2^3 \div 2^4$

6.2 Evaluate: $(3^2)^3$

6.3 Simplify: $5^{(-2)} \times 5^5$

6.4 A bacteria culture doubles every 2 hours. If there are initially 100 bacteria, how many will there be after 8 hours?

Solutions

6.1: $2^{(5+3-4)} = 2^4 = 16$

6.2: $3^{(2\times3)} = 3^6 = 729$

6.3: $5^{(-2+5)} = 5^3 = 125$

6.4: After 8 hours (4 doubling periods): $100 \times 2^4 = 1600$ bacteria

🧩 Chapter 7: Advanced Algebraic Word Problems

Problem-Solving Strategy

1. Identify what you need to find
2. Define variables clearly
3. Set up equations from given information
4. Solve systematically
5. Check your answer makes sense

Worked Example 7.1 - Age Problem

Problem: Sarah is 3 times as old as her brother Tom. In 5 years, Sarah will be twice as old as Tom. How old are they now?

Let: Tom's current age = x years, Sarah's current age = 3x years

In 5 years: Tom = x + 5, Sarah = 3x + 5

Equation: $3x + 5 = 2(x + 5)$

Solve: $3x + 5 = 2x + 10 \rightarrow x = 5$

Answer: Tom is 5 years old, Sarah is 15 years old

Check: In 5 years: Tom = 10, Sarah = 20. Indeed 20 = 2 × 10 ✓

Worked Example 7.2 - Mixture Problem

Problem: A chemist mixes a 30% acid solution with a 60% acid solution to make 500mL of 45% acid solution. How much of each solution is needed?

Let: x = mL of 30% solution, (500 - x) = mL of 60% solution

Equation: $0.30x + 0.60(500 - x) = 0.45(500)$

Expand: $0.30x + 300 - 0.60x = 225$

Solve: $-0.30x = -75 \rightarrow x = 250$

Answer: 250mL of 30% solution, 250mL of 60% solution

Practice Problems

7.1 The length of a rectangle is 5cm more than twice its width. If the perimeter is 46cm, find the dimensions.

7.2 A number is 7 more than another number. If their product is 120, find the numbers.

7.3 A cyclist travels 60km. For the first part at 15 km/h, then 20 km/h for the remainder. If the total time is 3.25 hours, how far did she travel at each speed?

7.4 The sum of three consecutive integers is 81. Find the integers.

Solutions

7.1: Width = 6cm, Length = 17cm

7.2: Let x and (x+7). x(x+7) = 120. Numbers are 8 and 15.

7.3: 30km at 15 km/h, 30km at 20 km/h

7.4: Let integers be x, x+1, x+2. Sum = 3x+3 = 81, so x = 26. Integers: 26, 27, 28

Chapter 8: Sequences and Patterns

Sequence Types

Arithmetic: $a, a+d, a+2d, \ldots$ (common difference d)

Geometric: $a, ar, ar^2, \ldots$ (common ratio r)

nth term: arithmetic: $a + (n-1)d$

nth term: geometric: $ar^{(n-1)}$

Worked Example 8.1 - Arithmetic Sequence

Find the 20th term of the sequence: 7, 11, 15, 19, ...

Step 1: Identify first term: $a = 7$

Step 2: Find common difference: $d = 11 - 7 = 4$

Step 3: Use formula: $T_n = a + (n-1)d$

Step 4: $T_{20} = 7 + (20-1)\times 4 = 7 + 76 = 83$

Answer: The 20th term is 83

Worked Example 8.2 - Geometric Sequence

Find the 6th term of the sequence: 2, 6, 18, 54, ...

Step 1: Identify first term: $a = 2$

Step 2: Find common ratio: $r = 6 \div 2 = 3$

Step 3: Use formula: $T_n = ar^{(n-1)}$

Step 4: $T_6 = 2 \times 3^{(6-1)} = 2 \times 3^5 = 2 \times 243 = 486$

Answer: The 6th term is 486

Practice Problems

8.1 Find the 15th term of: 5, 9, 13, 17, ...

8.2 Find the 5th term of: 3, 12, 48, 192, ...

8.3 The 8th term of an arithmetic sequence is 31 and the 12th term is 47. Find the first term.

8.4 A geometric sequence has first term 5 and common ratio 2. Find the sum of the first 6 terms.

Solutions

8.1: $a = 5$, $d = 4$. $T_{15} = 5 + 14\times 4 = 61$

8.2: $a = 3$, $r = 4$. $T_5 = 3\times 4^4 = 3\times 256 = 768$

8.3: Set up: $a + 7d = 31$, $a + 11d = 47$. Solving: $d = 4$, $a = 3$

8.4: Sum $= 5(2^6-1)/(2-1) = 5\times 63 = 315$

Chapter 9: Coordinate Geometry with Algebra

Key Formulas

Distance: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Midpoint: $M = ((x_1+x_2)/2, (y_1+y_2)/2)$

Gradient: $m = (y_2-y_1)/(x_2-x_1)$

Parallel lines: same gradient

Perpendicular lines: $m_1 \times m_2 = -1$

Worked Example 9.1

Find the equation of the line perpendicular to y = 2x + 3 passing through (4, 1):

- **Step 1:** Original gradient = 2
- **Step 2:** Perpendicular gradient = -1/2
- **Step 3:** Use point-gradient form: y - 1 = -1/2(x - 4)
- **Step 4:** Simplify: y - 1 = -x/2 + 2
- **Answer:** y = -x/2 + 3

Worked Example 9.2

A triangle has vertices A(1, 2), B(5, 6), and C(3, 8). Find the length of side AB:

- **Step 1:** Use distance formula
- **Step 2:** $AB = \sqrt{(5-1)^2 + (6-2)^2}$
- **Step 3:** $AB = \sqrt{16 + 16} = \sqrt{32}$
- **Step 4:** $AB = \sqrt{16\times2} = 4\sqrt{2}$
- **Answer:** $AB = 4\sqrt{2} \approx 5.66$ units

Practice Problems

9.1 Find the midpoint of the line segment joining (3, 7) and (9, 1)

9.2 Find the gradient of the line passing through (-2, 5) and (4, -1)

9.3 Find the equation of the line parallel to y = 3x - 2 passing through (2, 7)

9.4 A quadrilateral has vertices at (0, 0), (4, 0), (4, 3), and (0, 3). Find its perimeter.

Solutions

9.1: Midpoint = ((3+9)/2, (7+1)/2) = (6, 4)

9.2: $m = (-1-5)/(4-(-2)) = -6/6 = -1$

9.3: Same gradient (3): $y - 7 = 3(x - 2) \rightarrow y = 3x + 1$

9.4: Rectangle with sides 4 and 3. Perimeter = 2(4 + 3) = 14 units

Chapter 10: Advanced Real-World Applications

Case Study 10.1 - Business Optimisation

Problem: A company's profit P (in thousands £) is modelled by $P = -2x^2 + 16x - 24$, where x is the number of products (in hundreds) manufactured. Find the optimal production level.

Method: Complete the square or find vertex

Step 1: $P = -2(x^2 - 8x) - 24$

Step 2: $P = -2(x^2 - 8x + 16 - 16) - 24$

Step 3: $P = -2((x - 4)^2 - 16) - 24$

Step 4: $P = -2(x - 4)^2 + 32 - 24 = -2(x - 4)^2 + 8$

Answer: Maximum profit of £8000 when x = 4 (400 products)

Case Study 10.2 - Motion Analysis

Problem: A projectile's height h (metres) after t seconds is given by $h = -5t^2 + 30t + 10$. When does it reach maximum height and what is that height?

Step 1: Find vertex of parabola $h = -5t^2 + 30t + 10$

Step 2: $t = -b/(2a) = -30/(2\times-5) = 3$ seconds

Step 3: $h(3) = -5(9) + 30(3) + 10 = -45 + 90 + 10 = 55$ metres

Answer: Maximum height of 55 metres at t = 3 seconds

Challenge Problems

10.1 A rectangular garden is to be enclosed with 100m of fencing. Express the area A in terms of width w, and find the dimensions for maximum area.

10.2 The population of a town grows according to $P = 5000(1.03)^t$, where t is years after 2020. When will the population reach 6000?

10.3 A box is made from a 20cm × 15cm rectangle by cutting squares of side x cm from each corner and folding. Express the volume V in terms of x.

Solutions

10.1: If width = w, length = 50-w. $A = w(50-w) = 50w-w^2$. Maximum when w = 25m (square garden)

10.2: $6000 = 5000(1.03)^t \rightarrow 1.2 = (1.03)^t \rightarrow t \approx 6.1$ years (2026)

10.3: $V = x(20-2x)(15-2x) = x(300-70x+4x^2) = 4x^3-70x^2+300x$

Band 8 Practice Assessment

Band 8 Level Questions

Question 1: Solve for x: $3(x - 2) + 2(x + 1) = 4(x - 3) + 15$

Question 2: A quadratic function has zeros at x = 2 and x = -3, and passes through (0, 12). Find the function.

Question 3: The sum of two numbers is 15. The sum of their squares is 113. Find the numbers.

Question 4: A geometric sequence has first term 2 and each term is 3 times the previous term. What is the sum of the first 5 terms?

Question 5: Find the equation of the circle with centre (3, -2) that passes through the point (7, 1).

Detailed Solutions

Solution 1: $3x - 6 + 2x + 2 = 4x - 12 + 15 \rightarrow 5x - 4 = 4x + 3 \rightarrow x = 7$

Solution 2: $f(x) = a(x - 2)(x + 3)$. Using (0, 12): $12 = a(-2)(3) = -6a \rightarrow a = -2$. Therefore $f(x) = -2(x - 2)(x + 3) = -2x^2 - 2x + 12$

Solution 3: Let numbers be x and 15-x. $x^2 + (15-x)^2 = 113 \rightarrow 2x^2 - 30x + 225 = 113 \rightarrow x^2 - 15x + 56 = 0 \rightarrow (x-7)(x-8) = 0$. Numbers are 7 and 8.

Solution 4: Terms: 2, 6, 18, 54, 162. Sum = $2(3^5-1)/(3-1) = 2(242)/2 = 242$

Solution 5: Radius = $\sqrt{[(7-3)^2 + (1-(-2))^2]} = \sqrt{[16 + 9]} = 5$. Equation: $(x-3)^2 + (y+2)^2 = 25$

💡 Band 8 Success Strategies

Problem-Solving Approach

- **Read carefully:** Identify key information and what's required
- **Visualise:** Draw diagrams or graphs when helpful
- **Plan method:** Choose appropriate algebraic techniques
- **Work systematically:** Show clear steps in your working
- **Check answers:** Substitute back or use estimation

Common Band 8 Mistakes

- Algebraic manipulation errors
- Forgetting to check solutions in original equation
- Misinterpreting word problems
- Sign errors when manipulating inequalities
- Not simplifying final answers

Preparation Checklist

Core Skills

✓ Equation solving
✓ Factorisation
✓ Function notation
✓ Graph interpretation

Advanced Topics

✓ Systems of equations
✓ Quadratic functions
✓ Exponential growth
✓ Coordinate geometry

Applications

✓ Real-world modelling
✓ Optimisation problems
✓ Financial mathematics
✓ Motion analysis

🏆 Your Path to Band 8 Excellence

Mastering Band 8 algebra requires dedication, practice, and strategic thinking. The concepts in this guide represent the pinnacle of Year 9 algebraic understanding and will serve as the foundation for advanced mathematics.

Key Achievements:

- Advanced equation manipulation
- Complex problem-solving strategies
- Real-world application skills
- Mathematical communication

Next Steps:

- Regular practice with varied problems
- Focus on areas of weakness
- Attempt past NAPLAN questions
- Seek additional challenge problems

> "Excellence in algebra is not about memorising formulas—it's about developing the ability to think logically, reason systematically, and solve problems creatively."