Heuristics Handbook

Model Drawing, Assumption & Working-Back Methods

Master the Three Most Powerful PSLE Mathematics Problem-Solving Techniques

Visual Learning | Step-by-Step Solutions | PSLE Success

Contents Overview

What Are Mathematics Heuristics?

Mathematics heuristics are problem-solving strategies that students can use to tackle complex word problems systematically. Unlike rote memorisation of formulas, heuristics teach students **how to think** mathematically and approach unfamiliar problems with confidence.

Why Heuristics Matter for PSLE Success

- ✓ **Problem-solving confidence:** Students learn to break down complex problems
- ✓ **Higher-order thinking:** Develops reasoning and analytical skills
- ✓ **Examination advantage:** Many PSLE questions specifically test heuristic application
- ✓ **Foundation for secondary mathematics:** Essential for advanced problem-solving

Pólya's 4-Step Problem-Solving Process
1. Understand → 2. Plan → 3. Execute → 4. Review

The 12 Singapore Math Heuristics

To Give Representation

- Draw a diagram/model
- Draw a table
- Make a systematic list

To Make Calculated Guess

- Look for patterns
- Guess and check
- Make suppositions

To Go Through Process

- Act it out
- Work backwards
- Use before-after concept

To Change the Problem

- Restate the problem
- Simplify the problem
- Solve part of the problem

The Three Most Crucial Heuristics for PSLE

Model Drawing
Visual representation using bars and diagrams

Assumption Method
Strategic suppositions to simplify complex problems

Working Backwards
Starting from the end result to find initial values

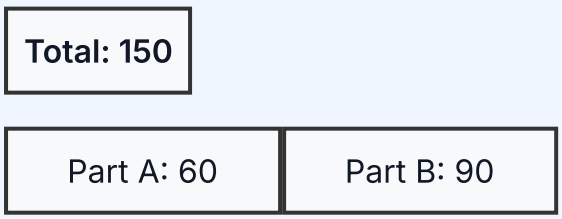
Model Drawing (Bar Modelling) Mastery

Model drawing, also known as bar modelling, is the most powerful visual heuristic in Singapore Mathematics. It transforms abstract word problems into concrete visual representations using rectangular bars.

Model Drawing Formula: Unknown Quantities → Visual Bars → Mathematical Relationships

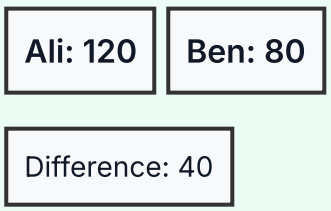
Types of Bar Models

1. Part-Whole Models



Used when the total and some parts are known, need to find unknown parts.

2. Comparison Models



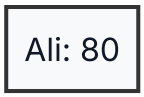
Used when comparing quantities, showing more/less relationships.

3. Ratio Models

4. Before-After Models

Before:

Used for ratio problems, showing proportional relationships.



After:

Used when quantities change over time or after operations.

5-Step Bar Modelling Approach

Step 1: Read and Understand

Identify what is given, what is unknown, and the relationships between quantities.

Step 2: Draw the Model

Create rectangles to represent the quantities, making sure proportions reflect the problem.

Step 3: Label Clearly

Add labels, question marks for unknowns, and any given numerical values.

Step 4: Calculate

Use the visual model to set up equations and solve mathematically.

Step 5: Verify

Check if your answer makes sense in the context of the original problem.

Worked Example: Model Drawing

Problem:
Sarah and Tom have 450 stickers altogether. Sarah has 3 times as many stickers as Tom. How many stickers does each person have?

Solution using Model Drawing:

Step 1: Understanding

- Total stickers = 450
- Sarah has 3 times Tom's amount
- Find: Individual amounts

Step 2 & 3: Draw and Label Model

Tom:

1 unit

Sarah:

1 unit	1 unit	1 unit

Total: 4 units = 450

Step 4: Calculate

```
4 units = 450
1 unit = 450 ÷ 4 = 112.5

Tom's stickers = 1 unit = 112.5
Sarah's stickers = 3 units = 112.5 × 3 = 337.5
```

Step 5: Verify

Check: 112.5 + 337.5 = 450 ✓
Check: 337.5 ÷ 112.5 = 3 ✓

💡 Assumption Method Mastery

The Assumption Method (also called Supposition Method) is a powerful technique where you make a strategic assumption about unknown values to simplify complex problems. This method is particularly effective for problems with multiple unknowns.

Assumption Method Formula: Make Assumption → Calculate Difference → Adjust for Reality

When to Use the Assumption Method

Perfect Scenarios

- Problems with two types of items
- Different values for each type
- Total quantity and total value given
- Need to find individual quantities

Key Indicators

- "Some... and some..." phrasing
- Different prices/values mentioned
- Total items and total cost given
- Mixed quantities (chickens/rabbits, etc.)

4-Step Assumption Method Process

Step 1: Make the Assumption

Assume all items are of one type (usually the simpler or cheaper option).

Step 2: Calculate the Assumed Total

Work out what the total value would be under your assumption.

Step 3: Find the Difference

Compare your assumed total with the actual total to find the discrepancy.

Step 4: Adjust for Reality

Use the difference to calculate how many items are actually of the other type.

Common Assumption Patterns

Money Problems

Pattern: Different coin/note values

Assumption: All items are smaller denomination

Animal Problems

Pattern: Different numbers of legs

Assumption: All animals are one type

Example: Chickens (2 legs) and rabbits (4 legs)

Example: $2 and $5 notes, total 20 notes worth $70

Transportation Problems

Pattern: Different capacities

Assumption: All vehicles are one type

Example: Cars (4 seats) and buses (30 seats)

Scoring Problems

Pattern: Different point values

Assumption: All questions worth same value

Example: Easy (2 points) and hard (5 points) questions

Worked Example: Assumption Method

Problem:
A farmer has chickens and cows. In total, there are 35 animals and 94 legs. How many chickens and how many cows are there?

Solution using Assumption Method:

Step 1: Make Assumption

Assume all 35 animals are chickens (2 legs each)

Step 2: Calculate Assumed Total

```
If all are chickens: 35 × 2 = 70 legs
```

Step 3: Find Difference

```
Actual legs − Assumed legs = 94 − 70 = 24 extra legs
```

Step 4: Adjust for Reality

```
Each cow has 2 more legs than chicken
Number of cows = 24 ÷ 2 = 12
Number of chickens = 35 − 12 = 23
```

Verification:

```
Chickens: 23 × 2 = 46 legs
Cows: 12 × 4 = 48 legs
Total: 46 + 48 = 94 legs ✓
Total animals: 23 + 12 = 35 ✓
```

Working Backwards Mastery

Working Backwards is a strategic heuristic used when you know the final result but need to find the initial value. Instead of following the problem forwards, you reverse the operations starting from the end.

Working Backwards Formula: Final State → Reverse Operations → Initial State

Perfect Scenarios for Working Backwards

Key Indicators

- "Finally had..." or "ended up with..."
- Sequence of operations described
- Final result is given
- Need to find starting amount
- Multiple steps in chronological order

Common Problem Types

- Money spent/received problems
- Items bought/sold/given away
- Multi-step calculations
- Age problems with time progression
- Fraction/percentage changes

Reverse Operations Guide

Forward → Reverse

Add (+)	→	Subtract (-)
Subtract (-)	→	Add (+)
Multiply (×)	→	Divide (÷)
Divide (÷)	→	Multiply (×)

Special Cases

Square (x^2)	→	Square root (√)
Double (×2)	→	Halve (÷2)
Increase by n	→	Decrease by n
Take away n	→	Add n

3-Step Working Backwards Process

Step 1: List All Operations in Order

Write down every operation mentioned in the problem from start to finish.

Step 2: Reverse the Operations

Starting from the final result, apply the reverse of each operation in reverse order.

Step 3: Verify Forward

Check your answer by working forwards through the original operations.

Worked Example: Working Backwards

Problem:
Liam had some money. He spent $15 on lunch, then doubled his remaining money at a game. After that, he gave $8 to his sister and finally had $22 left. How much money did Liam have initially?

Solution using Working Backwards:

Step 1: List Operations in Order

1. Started with unknown amount
2. Spent $15 on lunch (subtract 15)
3. Doubled remaining money (multiply by 2)
4. Gave $8 to sister (subtract 8)
5. Finally had $22

Step 2: Work Backwards

```
Final amount: $22
Before giving $8 to sister: $22 + $8 = $30
Before doubling: $30 ÷ 2 = $15
Before spending $15 on lunch: $15 + $15 = $30
```

Step 3: Verify Forward

```
Started with: $30
After lunch: $30 − $15 = $15
```

```
After doubling: $15 × 2 = $30
After giving sister: $30 − $8 = $22 ✓
```

Answer: Liam initially had $30.

Practice Problems

Model Drawing Practice

Problem 1: Amy and Ben have $180 altogether. Amy has $20 more than Ben. How much money does each person have?

Try drawing a comparison model to solve this problem.

Problem 2: In a school, the ratio of boys to girls is 3:4. There are 210 students in total. How many boys and girls are there?

Use a ratio model with units to solve this.

Assumption Method Practice

Problem 3: A parking lot has cars and motorcycles. There are 25 vehicles in total with 70 wheels. How many cars and motorcycles are there?

Assume all vehicles are motorcycles first.

Problem 4: Tom has $5 and $2 notes totalling $31. He has 8 notes altogether. How many of each type does he have?

Try assuming all notes are $2 notes first.

↺ Working Backwards Practice

Problem 5: Sarah had some stickers. She gave away 12 stickers, then bought 8 more. She then gave half of her remaining stickers to her friend and finally had 15 stickers left. How many stickers did Sarah have initially?

Work backwards from the final amount.

Problem 6: A number is multiplied by 3, then 7 is added. The result is divided by 2, and finally 5 is subtracted to get 13. What was the original number?

Reverse each operation step by step.

🔑 Practice Problem Solutions

Problem 1:
Ben: $80, Amy: $100

Problem 2:
Boys: 90, Girls: 120

Problem 3:
Cars: 10, Motorcycles: 15

Problem 4:
$5 notes: 3, $2 notes: 5

Problem 5:
Initially: 34 stickers

Problem 6:
Original number: 10

🎓 PSLE Examination Success Strategies

Quick Heuristic Selection Guide

Use Model Drawing when:
• Comparing quantities • Ratio problems • Part-whole relationships

Use Assumption Method when:
• Two types of items • Different values given • Total quantity and value known

Use Working Backwards when:
• Final result given • Multiple operations described • Need initial value

Time Management Tips

Read Carefully (30 seconds)
Identify key information and relationships

Choose Heuristic (15 seconds)
Quick decision based on problem type

Execute Solution (2-3 minutes)
Draw/calculate systematically

Verify Answer (30 seconds)
Quick check for reasonableness

Common Mistakes to Avoid

Model Drawing Errors
• Wrong proportions • Missing labels • Incorrect relationships

Assumption Method Errors
• Wrong assumption choice • Calculation mistakes • Forgetting to adjust

Working Backwards Errors
• Wrong reverse operations • Order confusion • No verification

Success Checklist

- [ ] Practised all three key heuristics regularly
- [ ] Can identify problem types quickly
- [ ] Draw clear, labelled diagrams
- [ ] Always verify answers
- [ ] Manage time effectively during practice
- [ ] Understand when to use each heuristic

Your Path to PSLE Mathematics Success

Model Drawing

Transform complex word problems into visual representations for clearer understanding

Assumption Method

Simplify challenging multi-variable problems through strategic assumptions

Working Backwards

Solve problems efficiently by starting from the known end result

Master these three heuristics and you'll have the tools to tackle any PSLE mathematics challenge with confidence!

- ✓ Comprehensive Coverage
- ✓ Worked Examples
- ✓ Practice Problems
- ✓ Success Strategies

Heuristics Handbook: Model Drawing, Assumption & Working-Back Methods

Your comprehensive guide to PSLE mathematics problem-solving success

Complete Guide | Practical Techniques | PSLE Focused