UK 11+ Exam Ratio & Proportion Problem Bank

Introduction

This comprehensive problem bank has been designed to support students preparing for the UK 11+ examination in the area of ratio and proportion. The problems are arranged in order of increasing difficulty, starting with fundamental concepts and progressing to complex real-world applications.

Ratio and proportion are essential mathematical concepts that appear frequently in 11+ examinations. Understanding these topics requires strong foundational knowledge in multiplication, division, fractions, and percentages. This problem bank provides extensive practice opportunities with detailed explanations and complete solutions.

Key Learning Objectives

- Understand ratio notation and terminology
- Simplify ratios and find equivalent ratios
- Share amounts in given ratios
- Solve direct and inverse proportion problems
- Apply scale factors to similar shapes
- Connect ratios to percentages and fractions
- Solve complex real-world applications

Chapter 1: Understanding Ratios

What is a Ratio?

A ratio compares two or more quantities and shows how much of one thing there is compared to another. Ratios are written using the colon notation (a:b) or as fractions.

> **Key Point: A ratio of 2:3 means "2 for every 3" or "2 parts to 3 parts"**

Worked Example 1.1

Question: In a bag of sweets, there are 6 red sweets and 9 blue sweets. What is the ratio of red sweets to blue sweets?

Solution:

Red sweets = 6

Blue sweets = 9

Ratio of red to blue = 6:9

We can simplify this by dividing both numbers by their highest common factor (HCF).

HCF of 6 and 9 = 3

6 ÷ 3 = 2

9 ÷ 3 = 3

Answer: The ratio of red sweets to blue sweets is 2:3

Practice Problems - Set 1

1.1

A recipe uses 4 cups of flour and 2 cups of sugar. What is the ratio of flour to sugar?

1.2

In a school, there are 15 boys and 12 girls in a class. What is the ratio of boys to girls?

1.3

A car travels 120 kilometres in 2 hours. What is the ratio of distance to time?

1.4

A garden has 8 roses and 6 tulips. What is the ratio of roses to tulips in its simplest form?

1.5

In a fruit bowl, there are 10 apples, 15 oranges, and 5 bananas. What is the ratio of apples to oranges to bananas?

Chapter 2: Equivalent Ratios and Simplifying

Understanding Equivalent Ratios

Equivalent ratios are ratios that represent the same relationship between quantities. They are created by multiplying or dividing both parts of a ratio by the same number.

Worked Example 2.1

Question: Find three equivalent ratios for 4:6

Solution:

Original ratio: 4:6

Multiply both parts by 2: (4×2):(6×2) = 8:12

Multiply both parts by 3: (4×3):(6×3) = 12:18

Divide both parts by 2: (4÷2):(6÷2) = 2:3

Answer: Three equivalent ratios are 8:12, 12:18, and 2:3

Simplifying Ratios

To simplify a ratio, we divide both parts by their highest common factor (HCF). The simplest form of a ratio has no common factors other than 1.

Worked Example 2.2

Question: Simplify the ratio 24:36

Solution:

Find the HCF of 24 and 36

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Common factors: 1, 2, 3, 4, 6, 12

HCF = 12

24 ÷ 12 = 2

36 ÷ 12 = 3

Answer: 24:36 simplifies to 2:3

Practice Problems - Set 2

2.1

Simplify the ratio 15:25

2.2

Write two equivalent ratios for 3:7

2.3

Simplify the ratio 18:24:30

2.4

Which of these ratios is equivalent to 4:5? A) 8:10 B) 12:14 C) 16:25 D) 20:25

2.5

A mixture contains ingredients in the ratio 14:21:35. Express this in its simplest form.

Chapter 3: Sharing Amounts in Given Ratios

Method for Sharing in Ratios

When sharing an amount in a given ratio, we:

1. Add up the parts of the ratio
2. Divide the total amount by the sum of the parts
3. Multiply each part by this value

Worked Example 3.1

Question: Share £240 in the ratio 3:5

Solution:

Total parts = 3 + 5 = 8

Value of each part = £240 ÷ 8 = £30

First share = 3 × £30 = £90

Second share = 5 × £30 = £150

Check: £90 + £150 = £240 ✓

Answer: The amounts are £90 and £150

Worked Example 3.2

Question: Three friends share 360 marbles in the ratio 4:5:6. How many marbles does each friend receive?

Solution:

Total parts = 4 + 5 + 6 = 15

Value of each part = 360 ÷ 15 = 24

First friend = 4 × 24 = 96 marbles

Second friend = 5 × 24 = 120 marbles

Third friend = 6 × 24 = 144 marbles

Check: 96 + 120 + 144 = 360 ✓

Answer: The friends receive 96, 120, and 144 marbles respectively

Practice Problems - Set 3

3.1

Share £180 in the ratio 2:3

3.2

Divide 450 sweets among three children in the ratio 2:3:4

3.3

A recipe calls for ingredients in the ratio 1:2:3. If the total weight is 480g, find the weight of each ingredient.

3.4

Two business partners share profits in the ratio 5:7. If the total profit is £3,600, how much does each partner receive?

3.5

A bag contains red, blue, and green balls in the ratio 3:4:5. If there are 144 balls in total, how many of each colour are there?

Chapter 4: Direct Proportion

Understanding Direct Proportion

Two quantities are in direct proportion when they increase or decrease at the same rate. If one quantity doubles, the other also doubles. If one quantity halves, the other also halves.

> **Key Point: In direct proportion, as one quantity increases, the other increases proportionally. The ratio between them remains constant.**

Worked Example 4.1

Question: If 5 pencils cost £3, how much do 8 pencils cost?

Solution:

This is direct proportion - more pencils cost more money

Method 1: Find the unit cost

Cost of 1 pencil = £3 ÷ 5 = £0.60

Cost of 8 pencils = 8 × £0.60 = £4.80

Method 2: Use proportion

5 pencils : £3 = 8 pencils : £x

x = (8 × £3) ÷ 5 = £24 ÷ 5 = £4.80

Answer: 8 pencils cost £4.80

Worked Example 4.2

Question: A car travels 180 kilometres in 3 hours. How far will it travel in 7 hours at the same speed?

Solution:

This is direct proportion - more time means more distance

Speed = 180 km ÷ 3 hours = 60 km/h

Distance in 7 hours = 60 km/h × 7 hours = 420 km

Answer: The car will travel 420 kilometres in 7 hours

Practice Problems - Set 4

4.1

If 6 books cost £18, how much do 10 books cost?

4.2

A recipe for 4 people uses 200g of flour. How much flour is needed for 10 people?

4.3

If 12 workers can build a wall in 8 days, how many days would it take 16 workers to build the same wall?

4.4

A machine produces 240 items in 6 hours. How many items can it produce in 9 hours?

4.5

If 3 metres of fabric cost £15, find the cost of 8 metres of the same fabric.

Chapter 5: Inverse Proportion

Understanding Inverse Proportion

Two quantities are in inverse proportion when one increases as the other decreases. If one quantity doubles, the other halves. If one quantity triples, the other becomes one-third.

> **Key Point: In inverse proportion, as one quantity increases, the other decreases proportionally. Their product remains constant.**

Worked Example 5.1

Question: 4 people can complete a job in 12 days. How many days would it take 6 people to complete the same job?

Solution:

This is inverse proportion - more people means less time needed

Total amount of work = 4 people × 12 days = 48 person-days

Time for 6 people = 48 person-days ÷ 6 people = 8 days

Answer: It would take 6 people 8 days to complete the job

Worked Example 5.2

Question: A car travelling at 60 km/h takes 4 hours to complete a journey. How long would the same journey take at 80 km/h?

Solution:

This is inverse proportion - higher speed means less time

Total distance = 60 km/h × 4 hours = 240 km

Time at 80 km/h = 240 km ÷ 80 km/h = 3 hours

Answer: The journey would take 3 hours at 80 km/h

Practice Problems - Set 5

5.1

8 machines can complete a task in 15 days. How many days would it take 12 machines to complete the same task?

5.2

A school has enough food for 200 students for 18 days. How many days would the food last if there were 240 students?

5.3

If 6 pumps can empty a tank in 4 hours, how long would it take 8 pumps to empty the same tank?

5.4

A cyclist travelling at 15 km/h takes 8 hours to complete a journey. How long would the journey take at 20 km/h?

5.5

12 cows have enough grass for 28 days. How many days would the grass last for 16 cows?

Chapter 6: Scale Factors and Similar Shapes

Understanding Scale Factors

A scale factor is the ratio between corresponding measurements of two similar shapes. It tells us how many times larger or smaller one shape is compared to another.

Worked Example 6.1

Question: A rectangle has dimensions 6 cm by 4 cm. It is enlarged by a scale factor of 3. What are the new dimensions?

Solution:

Original dimensions: 6 cm × 4 cm

Scale factor = 3

New length = 6 cm × 3 = 18 cm

New width = 4 cm × 3 = 12 cm

Answer: The new dimensions are 18 cm by 12 cm

Worked Example 6.2

Question: A map has a scale of 1:50,000. If the distance between two towns on the map is 8 cm, what is the actual distance?

Solution:

Scale 1:50,000 means 1 cm on the map represents 50,000 cm in reality

Map distance = 8 cm

Actual distance = 8 cm × 50,000 = 400,000 cm

Convert to kilometres: 400,000 cm = 4,000 m = 4 km

Answer: The actual distance is 4 kilometres

Practice Problems - Set 6

6.1

A triangle has sides of 5 cm, 7 cm, and 9 cm. If it is enlarged by a scale factor of 2.5, what are the new side lengths?

6.2

A photograph measuring 10 cm by 15 cm is enlarged so that the longer side becomes 24 cm. What is the scale factor?

6.3

On a map with scale 1:25,000, a lake appears to be 6 cm long. What is the actual length of the lake?

6.4

Two similar rectangles have areas in the ratio 4:9. What is the ratio of their corresponding sides?

6.5

A model car is built to a scale of 1:20. If the model is 15 cm long, what is the length of the real car?

Chapter 7: Percentages and Ratios

Converting Ratios to Percentages

To convert a ratio to percentages, we find what fraction each part represents of the total, then convert to percentages.

Worked Example 7.1

Question: In a class, the ratio of boys to girls is 3:2. What percentage of the class are boys?

Solution:

Ratio boys:girls = 3:2

Total parts = 3 + 2 = 5

Boys represent 3 parts out of 5 total parts

Fraction of boys = 3/5

Percentage of boys = (3/5) × 100% = 60%

Answer: 60% of the class are boys

Worked Example 7.2

Question: A mixture contains three ingredients in the ratio 5:3:2. What percentage of the mixture is each ingredient?

Solution:

Ratio = 5:3:2

Total parts = 5 + 3 + 2 = 10

First ingredient = (5/10) × 100% = 50%

Second ingredient = (3/10) × 100% = 30%

Third ingredient = (2/10) × 100% = 20%

Check: 50% + 30% + 20% = 100% ✓

Answer: The ingredients are 50%, 30%, and 20% respectively

Practice Problems - Set 7

7.1

The ratio of cats to dogs in a pet shop is 4:5. What percentage of the animals are cats?

7.2

A fruit drink contains orange juice and water in the ratio 2:3. What percentage of the drink is orange juice?

7.3

In a survey, the ratio of people who prefer tea to coffee to neither is 7:5:3. What percentage prefer tea?

7.4

A garden has flowers and vegetables in the ratio 3:7. If 15% of the garden is flowers, is this ratio correct?

7.5

The ratio of successful to unsuccessful job applications is 2:13. What percentage of applications are successful?

Chapter 8: Real-World Applications

Cooking and Recipes

Worked Example 8.1

Question: A recipe for 6 people uses 450g of flour, 300g of sugar, and 150g of butter. How much of each ingredient is needed for 10 people?

Solution:

This is direct proportion - more people need more ingredients

Scale factor = 10 people ÷ 6 people = 5/3

Flour needed = 450g × (5/3) = 450g × 5 ÷ 3 = 750g

Sugar needed = 300g × (5/3) = 300g × 5 ÷ 3 = 500g

Butter needed = 150g × (5/3) = 150g × 5 ÷ 3 = 250g

Answer: For 10 people: 750g flour, 500g sugar, 250g butter

Money and Finance

Worked Example 8.2

Question: Sarah and Tom invest money in a business in the ratio 4:3. The business makes a profit of £2,800. How much profit does each person receive?

Solution:

Investment ratio = 4:3

Total parts = 4 + 3 = 7

Value of each part = £2,800 ÷ 7 = £400

Sarah's share = 4 × £400 = £1,600

Tom's share = 3 × £400 = £1,200

Check: £1,600 + £1,200 = £2,800 ✓

Answer: Sarah receives £1,600 and Tom receives £1,200

Practice Problems - Set 8

8.1

A paint mixture requires red and blue paint in the ratio 5:2. How much blue paint is needed if 75 litres of red paint are used?

8.2

A school has 1,200 students. The ratio of students in Years 7, 8, and 9 is 5:4:3. How many students are in each year?

8.3

Two partners start a business investing £8,000 and £12,000 respectively. If the profit is £15,000, how should it be shared?

8.4

A concrete mixture uses cement, sand, and gravel in the ratio 1:2:4. How much sand is needed if 35 kg of cement is used?

8.5

A shop sells apples and oranges in the ratio 7:3. If 420 apples are sold, how many oranges are sold?

Chapter 9: Advanced Problem Solving

Complex Multi-Step Problems

Worked Example 9.1

Question: In a school, the ratio of teachers to students is 1:15. There are 480 people in the school in total. How many teachers and how many students are there?

Solution:

Ratio teachers:students = 1:15

Total parts = 1 + 15 = 16

Total people = 480

Value of each part = 480 ÷ 16 = 30

Number of teachers = 1 × 30 = 30

Number of students = 15 × 30 = 450

Check: 30 + 450 = 480 ✓

Answer: There are 30 teachers and 450 students

Worked Example 9.2

Question: A bag contains red, blue, and green balls in the ratio 4:3:2. After removing 6 red balls, the ratio becomes 2:3:2. How many balls were originally in the bag?

Solution:

Original ratio = 4:3:2

Let the common factor be x

Original balls: Red = 4x, Blue = 3x, Green = 2x

After removing 6 red balls: Red = 4x - 6, Blue = 3x, Green = 2x

New ratio = 2:3:2

So: (4x - 6):(3x):(2x) = 2:3:2

From the blue and green parts: 3x:2x = 3:2 ✓

From the red and blue parts: (4x - 6):3x = 2:3

Cross multiply: 3(4x - 6) = 2(3x)

12x - 18 = 6x

6x = 18

x = 3

Original balls: Red = 4×3 = 12, Blue = 3×3 = 9, Green = 2×3 = 6

Total = 12 + 9 + 6 = 27 balls

Answer: There were originally 27 balls in the bag

Practice Problems - Set 9

9.1

A father is 4 times as old as his son. In 20 years' time, the ratio of their ages will be 2:1. How old are they now?

9.2

Two numbers are in the ratio 3:4. If 5 is added to each number, the ratio becomes 4:5. Find the original numbers.

9.3

A rectangular field has its length and width in the ratio 5:3. If the perimeter is 160 metres, find the dimensions of the field.

9.4

Three friends have money in the ratio 2:3:4. After the first friend spends £12, the ratio becomes 1:3:4. How much money did each friend have originally?

9.5

A solution contains alcohol and water in the ratio 3:7. If 20 litres of water is added, the ratio becomes 3:11. How much alcohol was in the original solution?

Chapter 10: Challenge Problems

Advanced Applications

10.1

A car manufacturer produces cars in three colours: red, blue, and white in the ratio 5:3:2. If they produce 2,000 cars per month, and the demand changes so that they need to produce red, blue, and white cars in the ratio 4:5:1, how many more blue cars will they produce monthly?

10.2

A recipe calls for ingredients A, B, and C in the ratio 2:3:5. If the recipe is doubled, but ingredient A is increased by 50% more than required, what is the new ratio of ingredients?

10.3

In a school election, candidates A, B, and C receive votes in the ratio 5:4:3. If candidate A receives 40 more votes than candidate C, how many votes did each candidate receive?

10.4

A mixture of two types of coffee beans costs £8 per kg and £12 per kg respectively. If they are mixed in the ratio 3:2, what is the cost per kg of the mixture?

10.5

Two taps can fill a tank. The first tap fills the tank in 6 hours, and the second tap fills it in 9 hours. If both taps are opened together, in what ratio do they contribute to filling the tank?

Answer Key

Set 1 Answers

1.1 2:1 (flour to sugar)

1.2 5:4 (boys to girls)

1.3 60:1 (kilometres to hours)

1.4 4:3 (roses to tulips)

1.5 2:3:1 (apples to oranges to bananas)

Set 2 Answers

2.1 3:5

2.2 Possible answers: 6:14, 9:21, 15:35

2.3 3:4:5

2.4 A) 8:10

2.5 2:3:5

Set 3 Answers

3.1 £72 and £108

3.2 100, 150, and 200 sweets

3.3 80g, 160g, and 240g

3.4 £1,500 and £2,100

3.5 36 red, 48 blue, and 60 green balls

Set 4 Answers

4.1 £30

4.2 500g

4.3 6 days

4.4 360 items

4.5 £40

Set 5 Answers

5.1 10 days

5.2 15 days

5.3 3 hours

5.4 6 hours

5.5 21 days

Set 6 Answers

6.1 12.5 cm, 17.5 cm, and 22.5 cm

6.2 Scale factor = 1.6

6.3 1,500 metres (1.5 km)

6.4 2:3

6.5 300 cm (3 metres)

Set 7 Answers

7.1 44.4% (or 4/9)

7.2 40%

7.3 46.7% (or 7/15)

7.4 No, flowers should be 30%

7.5 13.3% (or 2/15)

Set 8 Answers

8.1 30 litres

8.2 Year 7: 500, Year 8: 400, Year 9: 300

8.3 £6,000 and £9,000

8.4 70 kg

8.5 180 oranges

Set 9 Answers

9.1 Son is 20 years old, Father is 80 years old

9.2 15 and 20

9.3 Length: 50m, Width: 30m

9.4 £24, £36, and £48

9.5 15 litres

Set 10 Answers

10.1 400 more blue cars

10.2 3:3:5

10.3 A: 200, B: 160, C: 120 votes

10.4 £9.60 per kg

10.5 3:2 (first tap to second tap)

Conclusion

This comprehensive problem bank has provided extensive practice in all aspects of ratio and proportion required for the UK 11+ examination. Students should work through the problems systematically, ensuring they understand each concept before progressing to more complex applications.

Remember that success in ratio and proportion problems requires:

- Understanding of the relationship between quantities
- Ability to recognise direct and inverse proportion
- Competence in simplifying and finding equivalent ratios
- Skill in sharing amounts proportionally
- Application of knowledge to real-world contexts

Regular practice with these types of problems will build confidence and proficiency in this important area of mathematics.